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Impact oscillators are an important class of non-smooth dynamical systems whose behaviour not only exhibits typical characteristics of smooth nonlinear systems, such as generic bifurcations, multiple solutions and chaos, but also displays new phenomena. Additionally, many physical situations of practical engineering relevance have limits incorporated in their dynamic response and therefore impacts cannot be avoided. Engineers must evaluate the dangers associated with these impacts to eliminate excessive loads, reduce wear, avoid fatigue and in some cases increase passenger comfort. A full understanding of the underlying mathematics increases our ability to account for and control impacts. A brief personal review is given of the literature relating to recent advances in dynamical systems theory pertaining to impacting systems and future areas of research are highlighted.

1. Introduction

An *impact oscillator* is the term used here to represent a system which is driven in some way and which also undergoes intermittent or a continuous sequence of contacts with motion limiting constraints. Even if the governing system is linear, the constraints introduce nonlinearity into the overall system. Investigations into the behaviour of nonlinear oscillators have proliferated throughout the scientific literature in recent years, boosted by the discovery of chaos and the complicated and sometimes fractal nature of the underlying mathematical structures. Impact oscillators incorporate typical nonlinear behaviour while additionally introducing discontinuities into the mathematics. At the same time, many physical systems of practical importance in engineering undergo impacts at motion limiting stops. Examples arise in marine engineering, where ships collide with fenders; in mechanical engineering, where gears rattle; plus many other examples.

Impacting systems are not new. Indeed, engineers have been faced with problems of wear and fatigue caused by repeated impacts for many years. To analyse the effects of impacts statistical methods have been developed and in some cases, novel devices incorporated to avoid inherent dangers. However, mathematical advances in smooth dynamical systems now provide a base for theories, experiments and numerical solutions to improve our fundamental understanding of impacting systems. In so doing two goals are achieved: on the one hand basic science is advanced with the analysis and classification of non-smooth systems, while on the other hand many situations of practical importance can now be viewed with greater understanding providing insight for further improvements.

One basic premise of many of the studies to date has been that the time spent during impact is small compared to the dynamic time. Furthermore, if no

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deformation of the two impacting surfaces occurs then a good approximation to the impacting process is one which involves an instantaneous reversal of velocities. A common technique in nonlinear dynamics is the study of the associated map of the system formed by either stroboscopically sampling the trajectory or, specifically for systems with impacts, via an examination of the system at each impact. It is the instantaneous change in velocity that produces discontinuities in the system (both in the map itself and, importantly, in its gradient) which can be considered to be a severe form of nonlinearity.

What are the consequences of these discontinuities for the theory of dynamical systems? As the parameters of a system are varied, if the qualitative nature of the dynamical behaviour changes, then in general we say a bifurcation has occurred. For smooth systems close to a bifurcation the use of centre manifold theory (Carr 1981) allows the dynamics to be reduced to a low (possibly one) dimensional manifold. On this centre manifold, coordinate transformations yield a normal form of the bifurcation which allows its classification. The same cannot be carried out in a straightforward manner for non-smooth systems with impact. It is now known that a bifurcation occurs for low velocity impacts, termed a *grazing* bifurcation, which is not the same as the simple bifurcations classified for smooth systems (Nordmark 1991; Foale & Bishop 1994). At these bifurcations the centre manifold does not exist but many of the other tools of nonlinear dynamical systems can still be applied, provided that especial attention is paid to those areas in which discontinuities occur. In particular the ideas of viewing a dynamical system in geometrical terms are also useful in impacting systems.

2. Engineering systems

Industrial situations which incorporate impacts are all around us. The difficulty lies in establishing those which can be viewed sensibly through our understanding of simple, low dimensional systems. This should not deter the scientist from tackling a problem of a seemingly complex nature as it is often via an industrially related problem that our understanding of basic science is advanced.

In the offshore environment impacting often occurs as a result of wave forcing. A typical scenario can be found in the early work by Thompson and his co-workers (Thompson & Ghaffari 1982). The wave driven motions of an articulated column, used for transferring oil from an offshore platform to a tanker, provided the motivation for studying bilinear systems with impact associated with a tether which becomes infinitely stiff when taut. In their work period-doubling bifurcations were noted which often resulted in the subsequent appearance of chaotic solutions. As in other works, experimental tests were performed to confirm numerical simulations. Further examples in marine technology are noted; where a ship is moored repeated contact with the fender is unavoidable (Lean 1971) while it is well known that rattling resonances can occur between the leg of the platform and pre-drilled piles during the docking procedure when installing new platforms offshore.

In mechanical engineering vibration, noise and wear is often attributed to impacts in machinery; these are possibly the most complicated to model and almost impossible to completely avoid. Early investigations evaluated the increase in impact forces due to clearances (Johnson 1958; Dubowsky & Freudenstein 1971; Dubowsky & Moening 1978; also see Haines 1979) and predicted the loss of contact

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between components (Earles & Seneviratne 1990). Further advances have also been made in the areas of gear rattle (Pfeiffer & Kunert 1990) and the avoidance of impact in journal bearings (Brindley *et al*. 1990).

Other aspects of engineering that have motivated related studies are the action of print hammers (Hendricks 1983), the behaviour of buildings under earthquake excitation (Housner 1963), tubes vibrating within a fluid (Connors 1982; Paidoussis & Li, 1992), as well as nonlinear vibration absorbers (Masri & Caughey 1966), to name but a few.

3. Mathematical and experimental studies

In terms of mathematical studies relating to impact oscillators emphasis has been placed either on conservative systems in which no energy loss occurs during dynamical motions between impacts, or dissipative systems in which energy losses can occur either during the motion between impacts (in the form of damping) or at impact, or indeed both. The particular form of the mathematical model and/or corresponding experimental apparatus varies between the various studies but all have a great deal in common. Investigators have considered theoretically mechanical spring–mass–damper models (Shaw & Holmes 1983*a*, *b*; Shaw 1985*a*, *b*) or merely the governing differential equations (Nordmark 1991; Foale & Bishop 1992). Though some research has been carried out to evaluate the response of impact oscillators under the action of random forcing (Davies 1980; Wood & Byrne 1981), for the most part the restriction of harmonic excitation has either been dictated by the motivating environment or chosen for simplicity. As always, the initial tasks are to locate periodic responses and assess their stability with particular attention paid to subsequent bifurcations and the global dynamics of the system (Whiston 1987). Software can either be written, or is available, to perform numerical simulations to determine the dynamic response once a system has been modelled (Peterka & Vacik 1992).

One subclass of systems investigated involves the behaviour of balls bouncing on a vibrating table (Veluswami & Crossley 1975; Veluswami *et al*. 1975; Holmes 1982; Everson 1986; Franaszek & Isomaki 1991). These works are complemented by those studies of a driven pendulum (either in the downward or inverted position) which impacts against a constraint (Moore & Shaw 1990; Shaw & Rand 1989; Bayley & Virgin 1993). While a third set of numerical and experimental studies considers the response of an elastic (or possibly an inelastic) beam (Moon & Shaw 1983; Shaw 1985*c*).

4. Outlook

Our increased understanding of dynamical systems theory has allowed us to advance considerably our knowledge of impacting systems. The three pronged attack of experimental studies, numerical simulations and a topological approach are all at such a level that further complex problems can now be attempted. High precision experimental apparatus coupled with computer control and data collection will enable future tests to confirm behaviour with even greater accuracy than previously achieved. Future numerical simulations should be able to reveal finer details of dynamical behaviour while a qualitative approach using topological methods will enable us to predict and classify behaviour for impacting systems within the wider class of non-smooth dynamical systems.

Future efforts will not only be directed towards the nature of discontinuous systems but also towards the ability of such models to capture the behaviour of engineering systems which undergo impacts. To date only systems with a low number of degrees of freedom (usually only one) have been considered and the challenge will be to show how these results and methods can be interpreted for more complex models.

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